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LINEAR FREQUENCY MODULATION OF OSCILLATORS

by

Louis M. Tozzi

March 1972

U S ARMY MATERIEL COMMAND
HARRY DIAMOND LABORATORIES
WASHINGTON D C 20438

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A technique is described for using varactors to obtain wideband frequency modulation of an oscillator. The modulation characteristics are linear with modulating voltage to within a fraction of a percent over bandwidths of up to 10 percent. Attainable linearity is a function of the bandwidth sought. The technique employed depends on the proper proportioning of a two-pole RF network in which the varactors are a component. It is shown that the higher frequency impedance pole moves linearly with varactor tuning voltage over the desired bandwidths. The technique is theoretically adaptable to any varactor power law. Graphs have been computed for varactors having capacitance of the form $C = a/\sqrt{V}$.

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CONTENTS

ILLUSTRATIONS

1. INTRODUCTION

An oscillator can be frequency modulated by means of a varactor diode coupled to its tank circuit. Varactor capacitance is related to the reverse bias voltage V by the relation

$$C = \frac{a}{(V)^{1/n}}$$

where n is generally between 2 and 3. If the varactor is simply connected across the tank circuit, the variations in frequency will not be linearly related to the modulating voltage, e.g.,

$$\omega = \frac{1}{\sqrt{LC}} = \frac{V^{1/2n}}{\sqrt{La}}$$

This approximates linearity only over a very small percentage bandwidth. The technique described here is capable of providing linear modulation over a large percentage bandwidth, say 10 or 15 percent, and for a reasonable range of the power law 1/n.

2. LINEAR FM WITH A TWO-POLE NETWORK

Wideband linear FM is possible using varactors in a two-complex-pole network. It is shown here that the higher frequency pole pair can be made to move linearly with varactor voltage if the circuit is properly proportioned.

Consider a circuit of the form shown in figure 1, where the active oscillator device is connected across C_1, and C_2 may be a pair of back-to-back varactors.

The input impedance is $\quad Z_{in} = \frac{\Delta''_{11}}{\Delta''}$

$$\text{where} \quad \Delta'' = -j\omega^{-3} \begin{vmatrix} \omega^2 C_1 - \frac{1}{L_1} & \frac{1}{L_1} & 0 \\ \frac{1}{L_1} & -\left(\frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}\right) & \frac{1}{L_2} \\ 0 & \frac{1}{L_2} & \omega^2 C_2 - \frac{1}{L_2} \end{vmatrix}$$

is the network admittance determinant; i.e., $I_i = Y_{ij}E_j$. At resonance, $Z_{in} = \infty$; hence, $\Delta'' = 0 = -j\omega^{-3}\Delta$. Now,

$$\Delta = \Delta^0 + \omega^2 C_2 \Delta_{33} \tag{2}$$

$$\text{where} \quad \Delta^0 = \begin{vmatrix} \omega^2 C_1 - \frac{1}{L_1} & \frac{1}{L_1} & 0 \\ \frac{1}{L_1} & -\left(\frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}\right) & \frac{1}{L_2} \\ 0 & \frac{1}{L_2} & -\frac{1}{L_2} \end{vmatrix}$$

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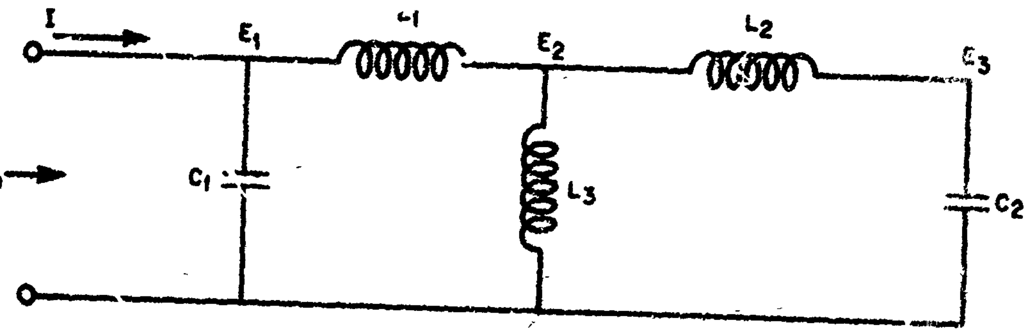

Figure 1. Two pole network.

and
$$\Delta_{33} = \begin{vmatrix} \omega^2 C_1 - \frac{1}{L_1} & \frac{1}{L_1} \\ \frac{1}{L_1} & -\left(\frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}\right) \end{vmatrix}$$

Letting $\Delta = 0$ in equation (2) gives

$$C_2 = -\frac{\Delta^0}{\omega^2 \Delta_{33}}$$

Let
$$\Delta^0 = \Delta^{0\prime} + \omega^2 C_1 \Delta^0_{11}$$

and
$$\Delta_{33} = \Delta'_{33} + \omega^2 C_1 \Delta_{1133}$$

where
$$\Delta^{0\prime} = \begin{vmatrix} -\frac{1}{L_1} & \frac{1}{L_1} & 0 \\ \frac{1}{L_1} & -\left(\frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}\right) & \frac{1}{L_2} \\ 0 & \frac{1}{L_2} & -\frac{1}{L_2} \end{vmatrix}$$

$$\Delta^0_{11} = \begin{vmatrix} -\left(\frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}\right) & \frac{1}{L_2} \\ \frac{1}{L_2} & -\frac{1}{L_2} \end{vmatrix}$$

$$\Delta'_{33} = \begin{vmatrix} -\frac{1}{L_1} & \frac{1}{L_1} \\ \frac{1}{L_1} & -\left(\frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}\right) \end{vmatrix}$$

$$\Delta_{1133} = -\left(\frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}\right)$$

then
$$C_2 = -\frac{\Delta^{0'} + \omega^2 C_1 \Delta^0_{11}}{\omega^2 (\Delta'_{33} + \omega^2 C_1 \Delta_{1133})}$$

or
$$C_2 = -\frac{1}{\omega^2} \cdot \frac{\Delta^0_{11}}{\Delta_{1133}} \cdot \frac{\left(\omega^2 + \frac{\Delta^{0'}}{C_1 \Delta^0_{11}}\right)}{\left(\omega^2 + \frac{\Delta'_{33}}{C_1 \Delta_{1133}}\right)}$$

Now we define the constants:

$$\gamma \equiv -\frac{\Delta_{1133}}{\Delta^0_{11}} = L_2 + \frac{L_1 L_3}{L_1 + L_3}$$

$$\alpha^2 \equiv -\frac{\Delta'_{33}}{C_1 \Delta_{1133}} = \frac{1}{C_1 \left(L_1 + \frac{L_2 L_3}{L_2 + L_3}\right)}$$

$$\beta^2 \equiv -\frac{\Delta^{0'}}{C_1 \Delta^0_{11}} = \frac{1}{C_1 (L_1 + L_{\cdot})}$$

(note that β/α is always less than unity);

and define
$$\omega_\alpha \equiv \frac{\omega}{\alpha}$$

Then
$$C_2 = \frac{1}{\gamma \alpha^2} \cdot \frac{\left(\omega_\alpha^2 - \frac{\beta^2}{\alpha^2}\right)}{\omega_\alpha^2 (\omega_\alpha - 1)(\omega_\alpha + 1)} = \frac{a}{V^{1/n}} \tag{3}$$

Raising this expression to the n-*th* power (n need not be an integer) gives

$$1 = \frac{V}{\gamma^n \alpha^{2n} a^n} \cdot \frac{\left(\omega_\alpha^2 - \frac{\beta^2}{\alpha^2}\right)^n}{\omega_\alpha^{2n} (\omega_\alpha - 1)^n (\omega_\alpha + 1)^n}$$

or
$$\omega_\alpha = \frac{1}{\gamma^n \alpha^{2n} a^n} \cdot \frac{V\left(\omega_\alpha^2 - \frac{\beta^2}{\alpha^2}\right)^n}{\omega_\alpha^{2n} (\omega_\alpha - 1)^{n-1} (\omega_\alpha + 1)^n} + 1$$

This is a straight line in V if

$$A(\omega_\alpha) \equiv \frac{\left(\omega_\alpha^2 - \frac{\beta^2}{\alpha^2}\right)^n}{\omega_\alpha^{2n} (\omega_\alpha - 1)^{n-1} (\omega_\alpha + 1)^n}$$

is constant in some region of ω_α. This is indeed the case for an appropriate choice of β^2/α^2 as shown in figure 2. Curves of this general form can be obtained for any value of n, but the desired value of the ratio β/α will differ for a different n. The values of $A(\omega)$ for values of ω_α between β/α and one are not shown in figure 2. In this region, $A(\omega)$ will be negative when n is an even integer, positive when n is an odd integer, and complex when n is not an integer.

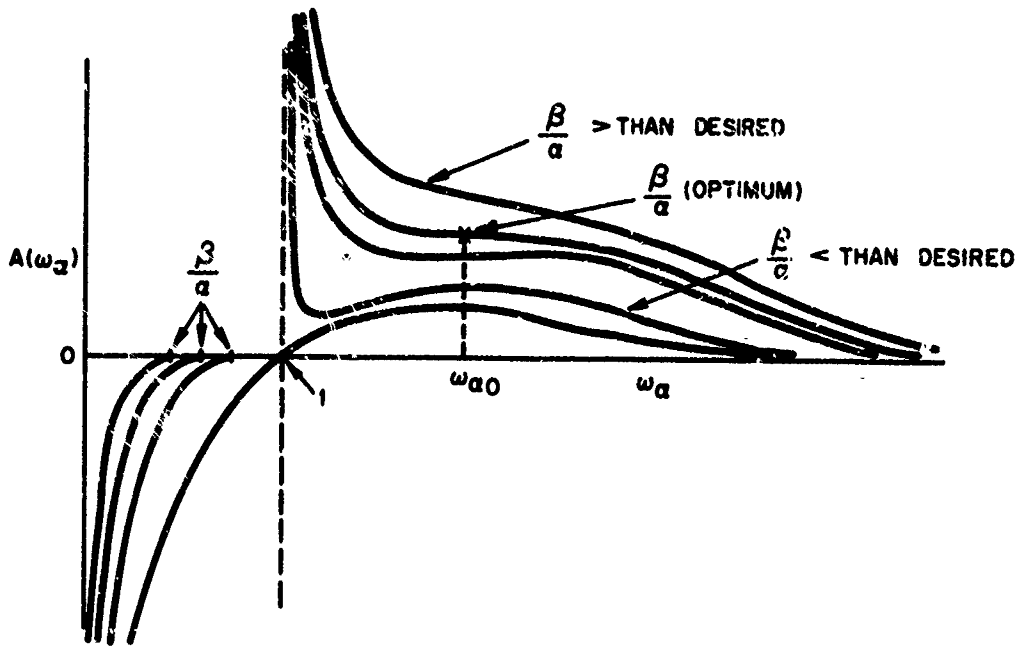

Figure 2. $A(\omega_\alpha)$ versus ω_α.

The appropriate region is the flat portion for $\omega_\alpha > 1$, and may be located analytically by setting $\frac{dA}{d\omega_\alpha} = 0$ and $\frac{d^2A}{d\omega_\alpha^2} = 0$ and solving simultaneously for β/α and ω_α. The region above one is the possible range of the upper pole and the region between zero and β/α is the possible range of the lower pole. This statement will become evident by examination of equation (5) in section 3 of this report. There can be no pole between β/α and one, unless C_2 is negative, as seen from equation (3). If it can be arranged through control of feedback that the oscillator operates in the region above one, the desired linear modulation can be achieved.

3. DETERMINATION OF MODULATION SENSITIVITY

Using equation (3) and defining

$$X_\alpha^2 = \frac{1}{C_2\gamma\alpha^2} = \frac{V^{1/n}}{a\gamma\alpha^2} \tag{4}$$

we obtain
$$\omega_\alpha^4 - (1 + X_\alpha^2)\omega_\alpha^2 + X_\alpha^2\frac{\beta^2}{\alpha^2} = 0$$

Solving for ω_α^2 gives

$$\omega_\alpha^2 = \frac{1}{2}\left[(1 + X_\alpha^2) \pm \left((1 + X_\alpha^2)^2 - 4X_\alpha^2\frac{\beta^2}{\alpha^2}\right)^{1/2}\right] \tag{5}$$

Now, for simplicity, let us assume an inverse square law varactor. Then from equation (4),

$$X_\alpha^4 = \frac{V}{a^2\gamma^2\alpha^4}$$

and defining $z = X_\alpha^4$, then

$$\omega_\alpha^2 = \frac{1}{2}\left\{1 + \sqrt{z} \pm \left[(1 + \sqrt{z})^2 - 4\sqrt{z}\frac{\beta^2}{\alpha^2}\right]^{1/2}\right\}$$

Differentiating with respect to z gives the normalized modulation sensitivity.

$$\frac{d\omega_\alpha}{dz} = \frac{\frac{1}{\sqrt{z}}\left[1 \pm \frac{(1+\sqrt{z}) - 2\frac{\beta^2}{\alpha^2}}{\left[(1+\sqrt{z})^2 - 4\sqrt{z}\frac{\beta^2}{\alpha^2}\right]^{1/2}}\right]}{4\sqrt{2}\left\{1 + \sqrt{z} \pm \left[(1+\sqrt{z})^2 - 4\sqrt{z}\frac{\beta^2}{\alpha^2}\right]^{1/2}\right\}^{1/2}} \tag{6}$$

Recalling the definition of ω_α and z gives

$$\frac{d\omega}{dV} = \frac{1}{a^2\gamma^2\alpha^3}\frac{d\omega_\alpha}{dz} \tag{7}$$

which is the modulation sensitivity in radians per volt. Equation (6) is plotted in figure 3 for various values of the parameter β^2/α^2 and $n = 2$. Figure 4 is a plot of equation (5) for the same conditions. With these curves, the circuit parameters are defined.

4. DETERMINATION OF CIRCUIT PARAMETERS

The circuit shown in figure 1 has five components, three of which can be determined if the remaining two are specified. Assume we wish to design a circuit to operate at the center angular frequency ω_0. From figure 3, we choose an operating curve for the best linearity corresponding to a given departure from linearity over a desired bandwidth. As an example, choose $\beta^2/\alpha^2 = 0.876$. The center of this flat region corresponds to an $X_\alpha^4 = 0.460$. From figure 4, we find the corresponding value of $\omega_{\alpha 0}$.

Now

$$\omega_0 = \omega_{\alpha 0}\alpha$$

hence, α is determined. Since β^2/α^2 has been chosen, β is also defined. If we choose a varactor, a is known, and from the useful voltage range choose V_0.

Then from

$$X_\alpha^4 = \frac{V_0}{a^2\gamma^2\alpha^4}$$

determine γ. The constants α, β, and γ are thus determined. From the defining equations for α, β, and γ, one can obtain

$$L_3^2 = \frac{\gamma}{C_1\beta^2}\left(\frac{1}{\beta^2/\alpha^2} - 1\right)$$

$$L_1 = \frac{1}{C_1\beta^2} - L_3$$

$$L_2 = \gamma - \frac{L_1L_3}{L_1 + L_3}$$

Hence, if C_1 is specified, L_1, L_2, and L_3 can be computed.

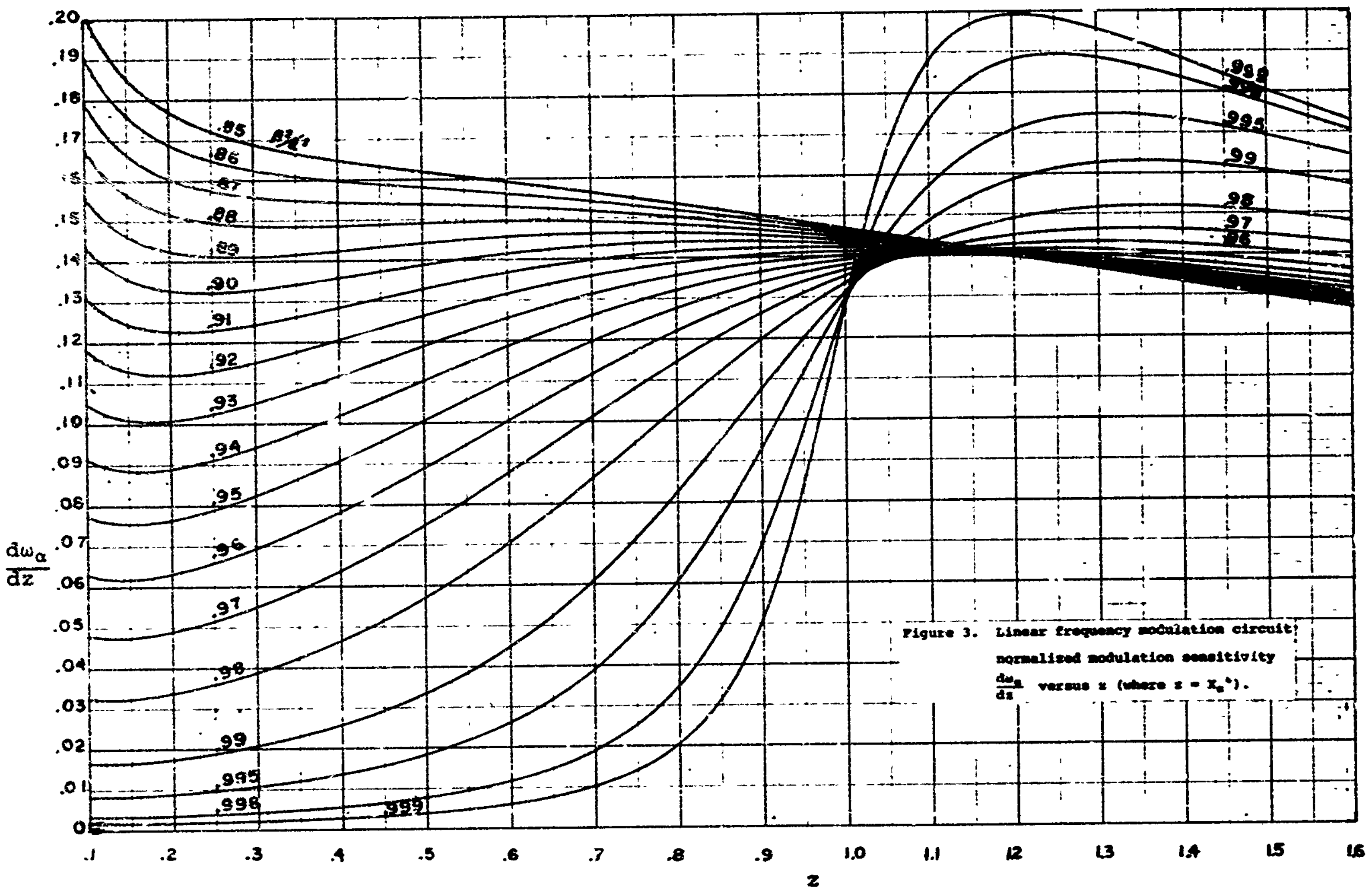

Figure 3. Linear frequency modulation circuit normalized modulation sensitivity $\frac{d\omega_a}{dz}$ versus z (where $z = X_a^*$).

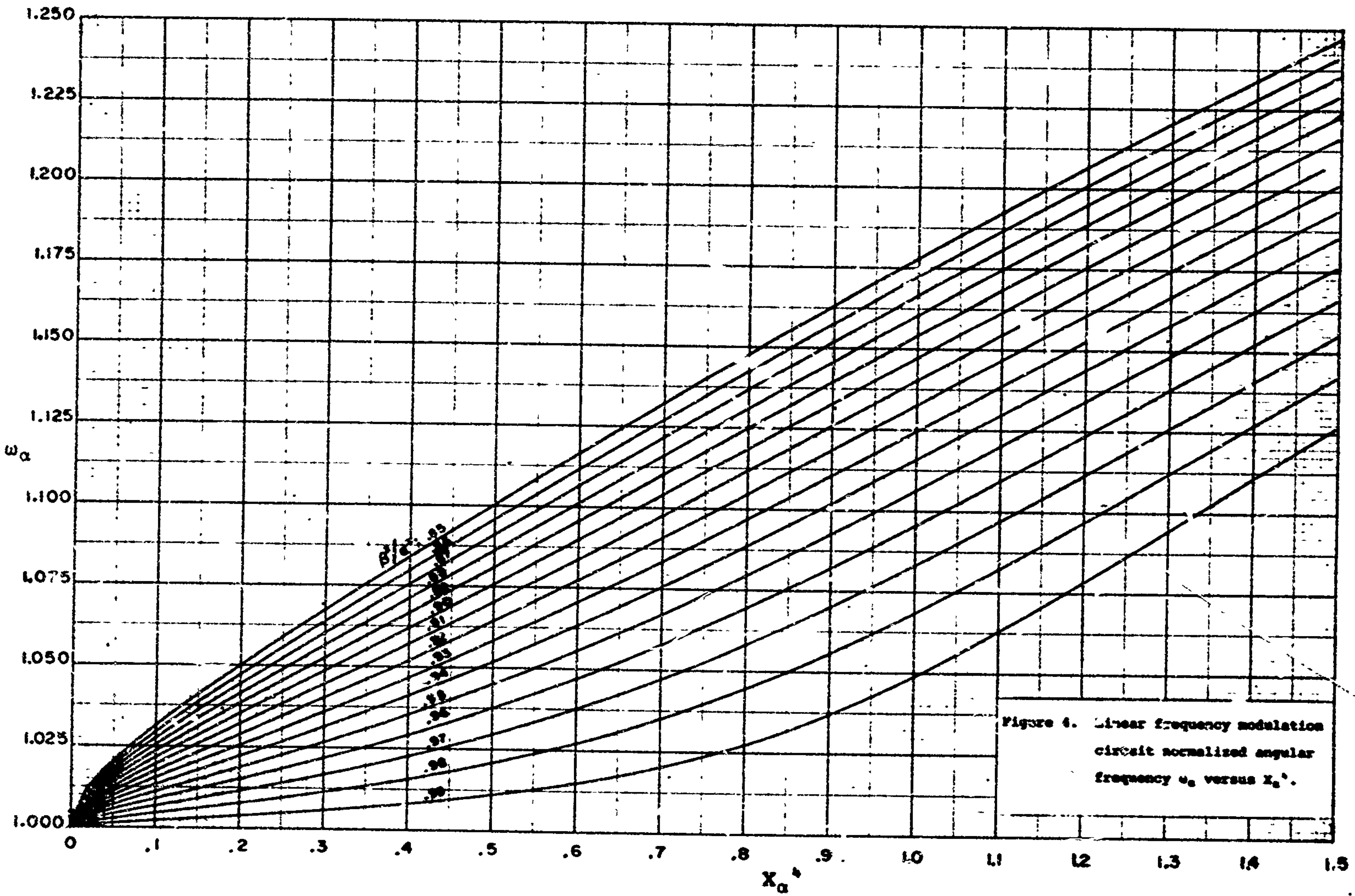

Figure 4. Linear frequency modulation circuit normalized angular frequency ω_a versus X_a^4.

Alternate relationships can be derived and graphed to provide a set of design curves that will allow a graphical solution. These are as follows:

$$\frac{L_1}{L_3} = \frac{\frac{\beta^2}{\alpha^2} - \frac{L_2}{L_2 + L_3}}{1 - \frac{\beta^2}{\alpha^2}} \tag{8}$$

$$L_1 + L_3 = \omega_{\alpha 0}{}^2 \frac{\alpha^2}{\beta^2} \frac{1}{C_1 \omega_0{}^2} \tag{9}$$

$$\frac{L_3{}^2}{L_1 + L_3} = \left(\frac{\alpha^2}{\beta^2} - 1\right) \frac{\left(\omega_{\alpha 0}{}^2 \quad \frac{\beta^2}{\alpha^2}\right)}{(\omega_{\alpha 0}{}^2 - 1)} \frac{1}{C_2 \omega_0{}^2} \tag{10}$$

where ω_0 is the desired design center angular frequency and

$$\omega_{\alpha 0} = \frac{\omega_0}{\alpha}$$

Using equation (8), figures 5, 7, 9, 11, 13, and 15 relate L_1 to L_3 with L_2 as a parameter for several values of β^2/α^2; figures 6, 8, 10, 12, 14, and 16 show two families (based on eqs 9 and 10) relating L_1 to L_3 with $C_1\omega_0{}^2$ and $C_2\omega_0{}^2$, respectively, as parameters. The constant k found in these figures is a scale factor chosen to fit the problem being solved. The values of β^2/α^2 and the design center value $\omega_{\alpha 0}$ for which the curves are valid are indicated on the respective graphs. These graphs, together with figures 3 and 4, provide compiled design information for the network. With all these graphs, an inverse square-root-varactor law (n = 2) has been assumed.

5. FURTHER OBSERVATIONS ON THIS NETWORK

From the defining equations, the quantities α and β can be shown to be

$$\alpha = \frac{1}{\left[C_1\left(L_1 + \frac{L_2 L_3}{L_2 + L_3}\right)\right]^{1/2}}$$

and

$$\beta = \frac{1}{[C_1(L_1 + L_3)]^{1/2}}$$

Hence, α is the angular frequency corresponding to $C_2 \to \infty$ (short circuit), and β is the angular frequency when $C_2 = 0$ (open circuit).

The coefficient of coupling (k) as normally defined can be seen to be related to the ratio β^2/α^2 by

$$\frac{\beta^2}{\alpha^2} = 1 - k^2 = \frac{L_1 + \frac{L_2 L_3}{L_2 + L_3}}{L_1 + L_3}$$

or

$$k = \frac{L_3}{[(L_1 + L_3)(L_2 + L_3)]^{1/2}}$$

A more significant observation can be made concerning the quantity X defined by equation (4).

$$X_\alpha{}^2 = \frac{1}{C_2\gamma\alpha^2}$$

Earlier, we determined the parallel resonances of the network from the poles of equation (1). A series resonance exists corresponding to the zeroes of equation (1), or $\Delta_{11}'' = 0$, where

$$\Delta_{11}'' = -\omega^{-2}\begin{vmatrix} -\left(\frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}\right) & \frac{1}{L_2} \\ \frac{1}{L_2} & \omega^2 C_2 - \frac{1}{L_2} \end{vmatrix} = -\omega^{-2}\Delta_{11}$$

Now, $\Delta_{11} = \Delta_{11}^{0} + \omega^2 C_2 \Delta_{1133} = 0$ for series resonance, and

$$\omega_{series}{}^2 = -\frac{\Delta_{11}^{0}}{C_2\Delta_{1133}}$$

Using the definition of γ gives

$$\omega_{series}{}^2 = \frac{1}{C_2\gamma}$$

X_α is then the normalized series resonant angular frequency

$$X_\alpha{}^2 = \frac{\omega_{series}{}^2}{\alpha^2}$$

Since the poles and zeroes must alternate on the $j\omega$ axis (also from eq 5), this series resonance must lie between the two parallel resonance points. Figure 4 shows the relationship between the desired resonance at ω_α and X_α. Clearly, the series resonance is closer to the parallel resonance as the frequency is increased. A consequence of this is that the resistance in the circuit, which has been neglected in the analysis, produces a greater loading at the higher frequency. This must be allowed for in the design of the oscillator if the amplitude is to be constant over the frequency range.

6. RF VOLTAGE ACROSS VARACTOR

The RF voltage across the varactor (C_2 in fig. 1) in terms of the network determinant is

$$E_3 = I\,\frac{\Delta_{13}''}{\Delta''}$$

and the input voltage is

$$E_1 = I\,\frac{\Delta_{11}''}{\Delta''}$$

The ratio E_3/E_1, after some manipulation, can be shown to be

$$\frac{E_3}{E_1} = \frac{1}{-\omega^2 C_2\left(\frac{1}{L_1} + \frac{L_2 + L_3}{L_2 L_3}\right)L_1 L_2 + \frac{L_1}{L_3} + 1}$$

With further manipulation, and expressing the ratio in terms of the quantity X (previously defined by eq 4) gives

$$\frac{E_3}{E_1} = \frac{-L_3}{L_1 + L_3} \cdot \frac{X}{\omega^2 - X^2}$$

This may be further expressed in terms of the constants α, β, and γ as

$$\frac{E_3}{E_1} = - \frac{X_\alpha{}^2}{\omega_\alpha{}^2 - X_\alpha{}^2} \left[C_1 \gamma \beta^2 \left(\frac{\alpha^2}{\beta^2} - 1\right)\right]^{1/2}$$

Now, ω_α is related to X_α by equation (5), and, as noted earlier, $\omega^2 - X^2$ gets smaller as X is increased; hence, the varactor RF voltage rises as the frequency is raised.

Using equation (5) to relate ω_α to X_α gives

$$\frac{E_3}{E_1} = \frac{-2X_\alpha{}^2 \left[C_1 \gamma \beta^2 \left(\frac{\alpha^2}{\beta^2} - 1\right)\right]^{1/2}}{1 - X_\alpha{}^2 \pm \left[(1 + X_\alpha{}^2)^2 - 4X_\alpha{}^2 \frac{\beta^2}{\alpha^2}\right]^{1/2}}$$

7. CONCLUSIONS

Relatively large percentage bandwidths can be achieved with good linearity. For example, choosing from figures 3 and 4 the curve $\beta^2/\alpha^2 = 0.877$, a center operating point of $X_\alpha{}^4 = 0.5$, and varying $X_\alpha{}^4$ from 0.3 to 0.7 gives a peak-to-peak variation in modulation sensitivity from 0.1503 to 0.1510, or 0.46 percent. The corresponding bandwidth is from 1.057 to 1.118, or 5.61 percent. Moving to the curve $\beta^2/\alpha^2 = 0.88$ and ranging in $X_\alpha{}^4$ from 0.25 to 0.8 gives 0.67-percent variation in modulation sensitivity with greater than 7.5-percent bandwidth. Greater bandwidths can be obtained with correspondingly greater departures from linearity.

Practical circuits have been built and operated in the vicinity of 30, 150, and 700 MHz which have confirmed the theoretical predictions. In practical high-frequency circuits, the inductor L_2 may be the lead inductance in the varactors. Other equivalent networks can be substituted that will yield the same results--e.g., the equivalent π, transformer coupling, etc. Since the network is not of canonic form, one of the two series inductors may be eliminated if practical considerations permit. Linear modulation can also be achieved with capacitive coupling between the meshes, and where some or all of the elements are distributed. New equations to define the circuit behavior need to be derived, however.

A portion of this analysis has been carried out with the varactor power law n = 2. Other values of n corresponding to more practical varactors can be used with essentially the same results. Varactors with n = 2.27 have been used with good results. For exact design, however, new curves would have to be generated. The curves presented are suitable for practical designs with some trimming of the final circuit, which is a reasonable thing to do at high frequencies.

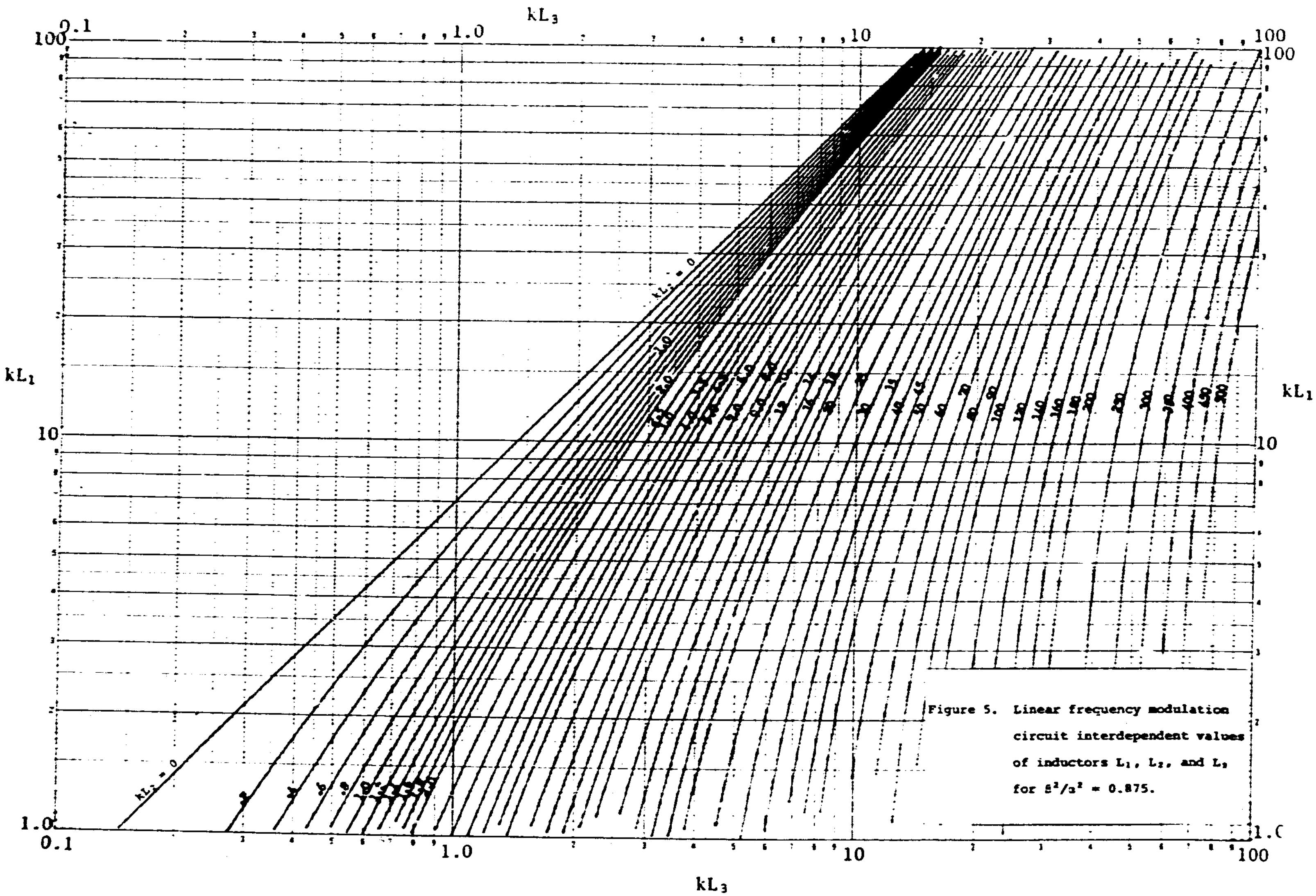

Figure 5. Linear frequency modulation circuit interdependent values of inductors L_1, L_2, and L_3 for $\delta^2/a^2 = 0.875$.

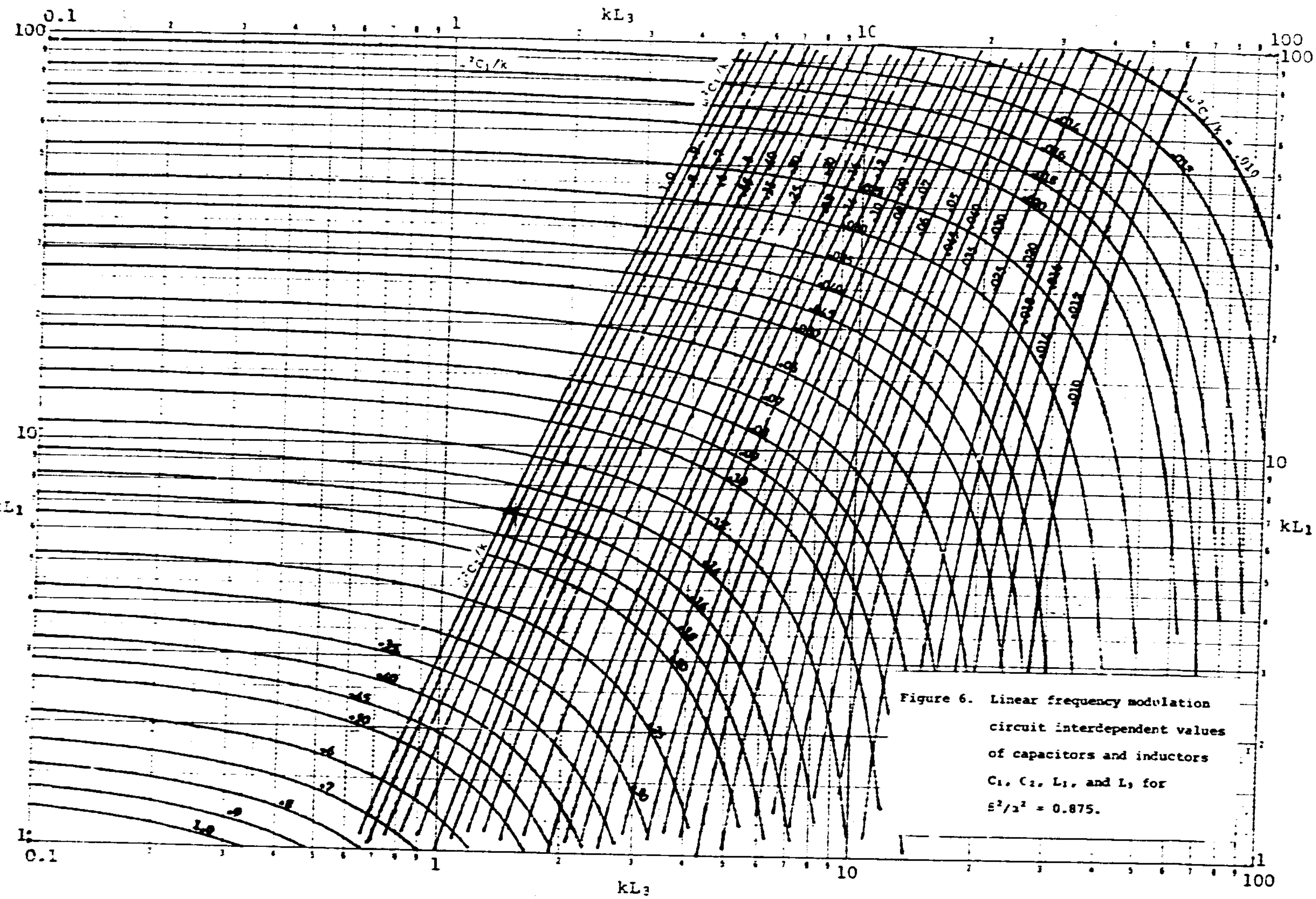

Figure 6. Linear frequency modulation circuit interdependent values of capacitors and inductors C_1, C_2, L_1, and L_3 for $f^2/f^2 = 0.875$.

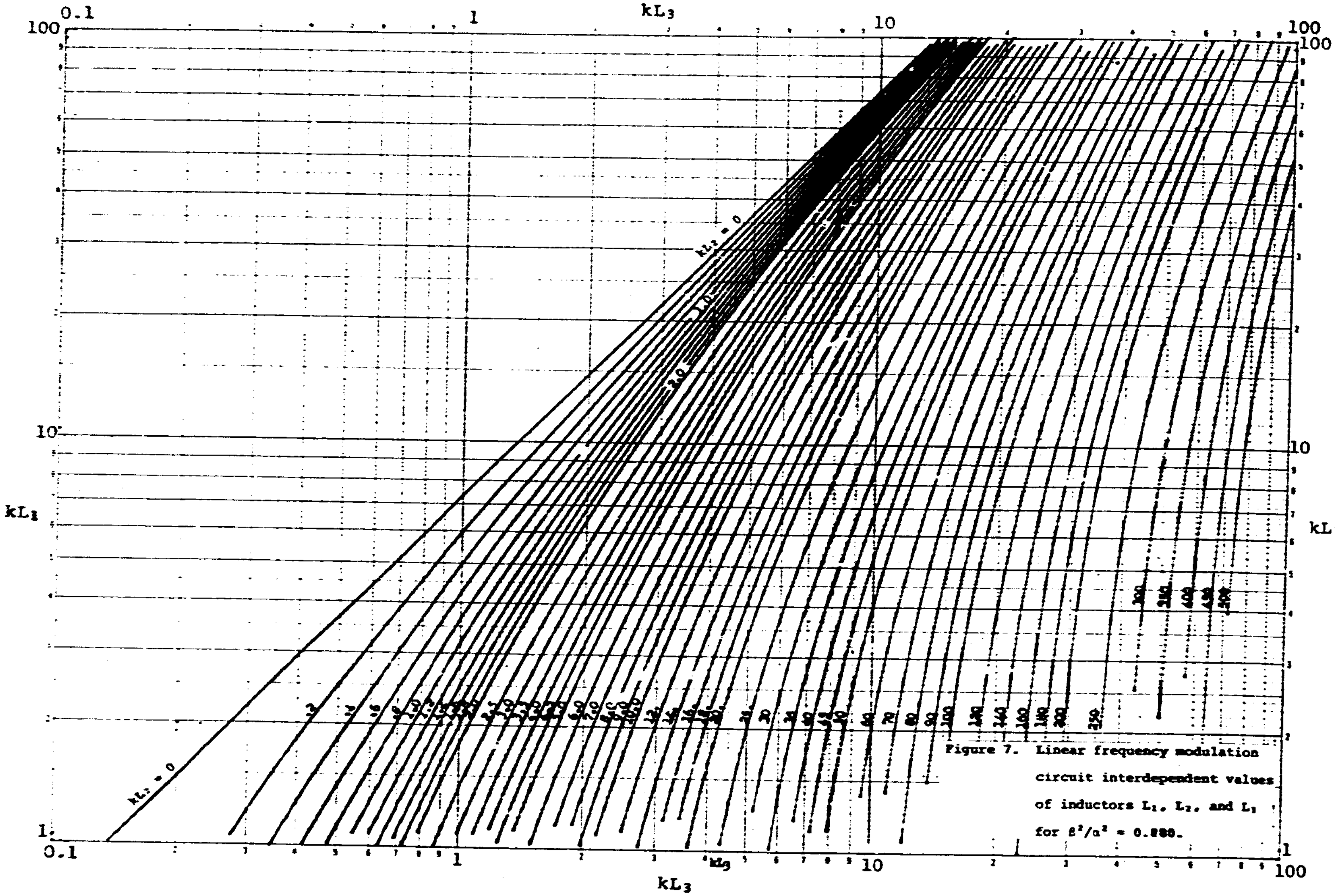

Figure 7. Linear frequency modulation circuit interdependent values of inductors L_1, L_2, and L_3 for $\beta^2/\alpha^2 = 0.888$.

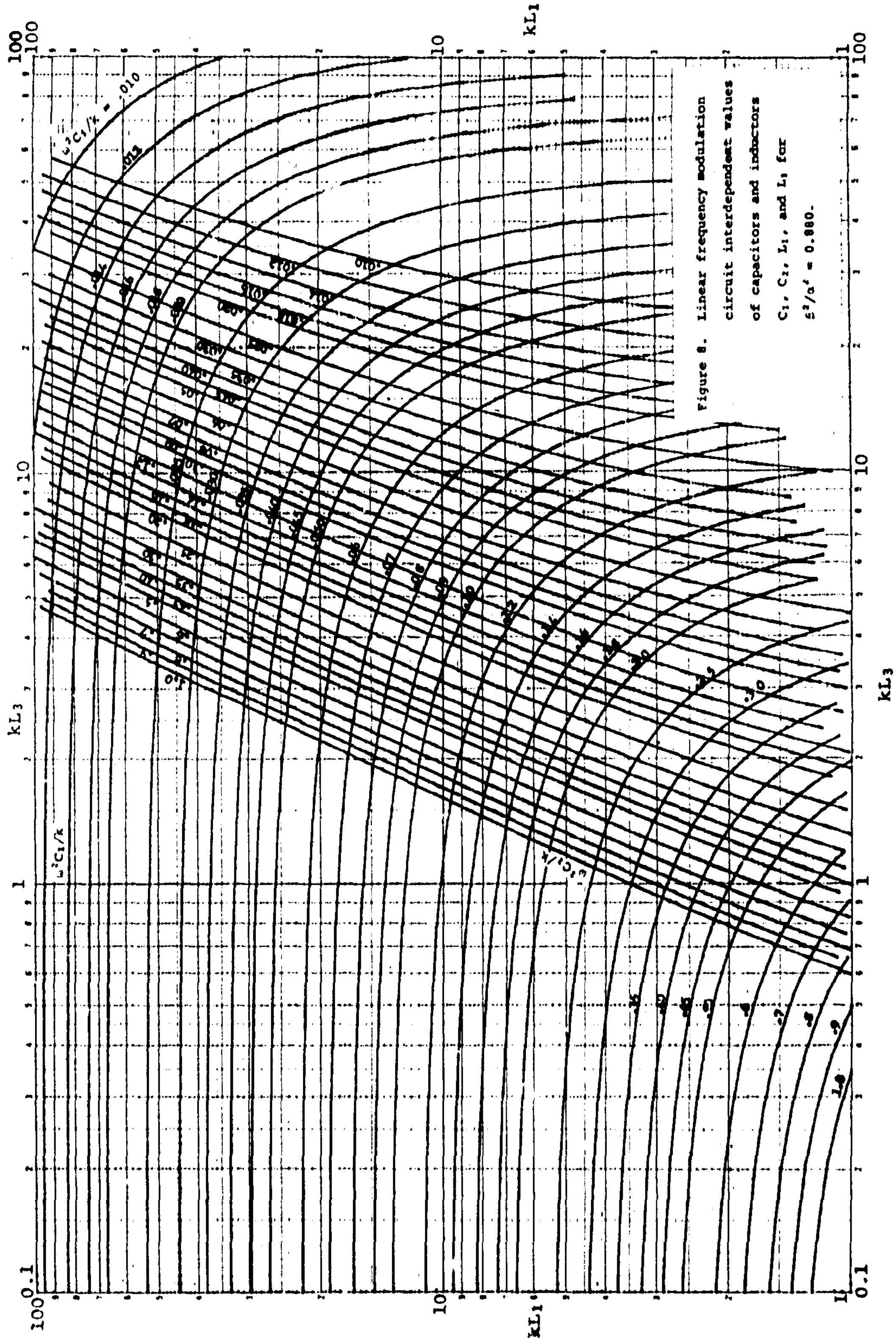

Figure 8. Linear frequency modulation circuit interdependent values of capacitors and inductors C_1, C_2, L_1, and L_3 for $f^2/a^2 = 0.880$.

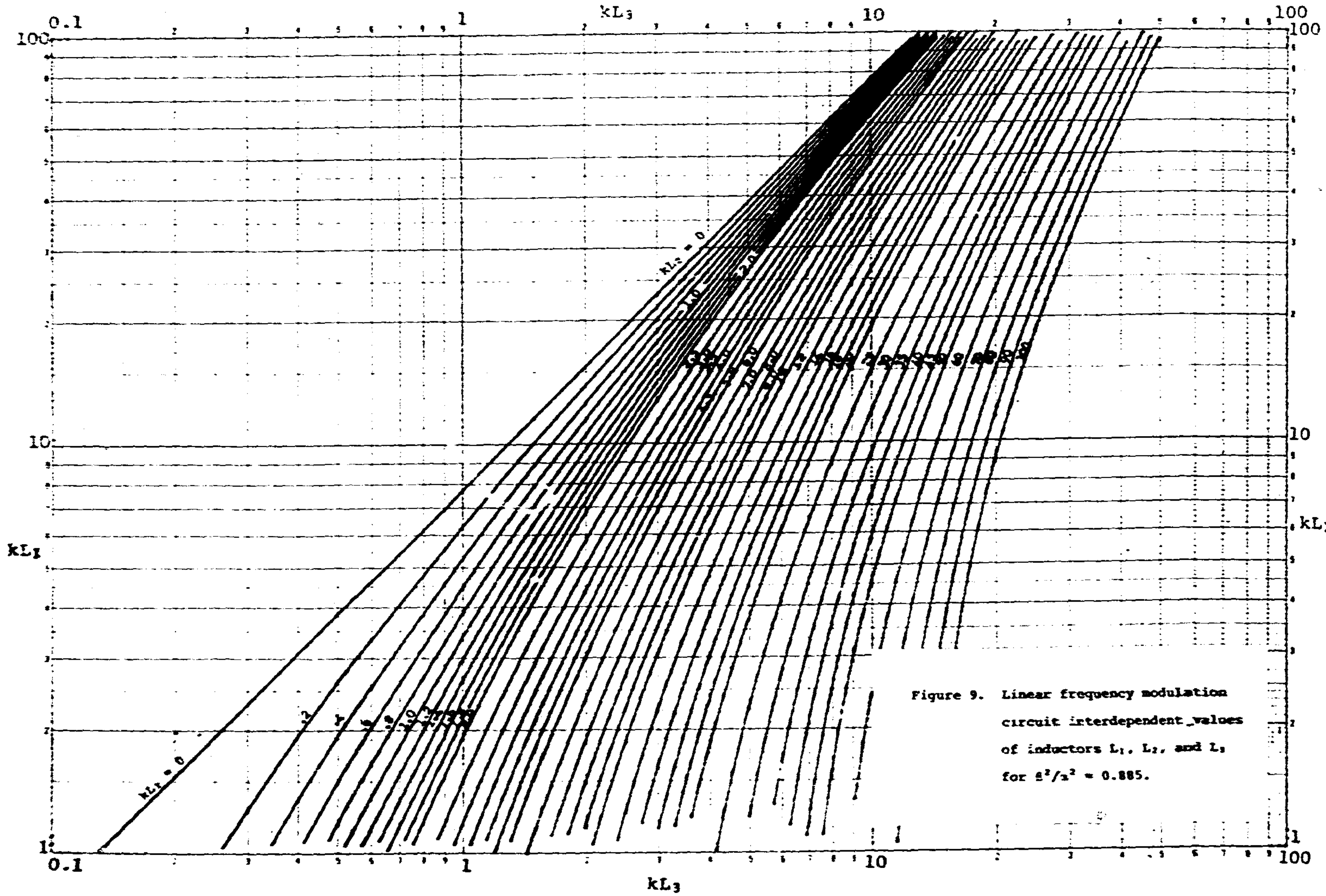

Figure 9. Linear frequency modulation circuit interdependent values of inductors L_1, L_2, and L_3 for $f^2/s^2 = 0.885$.

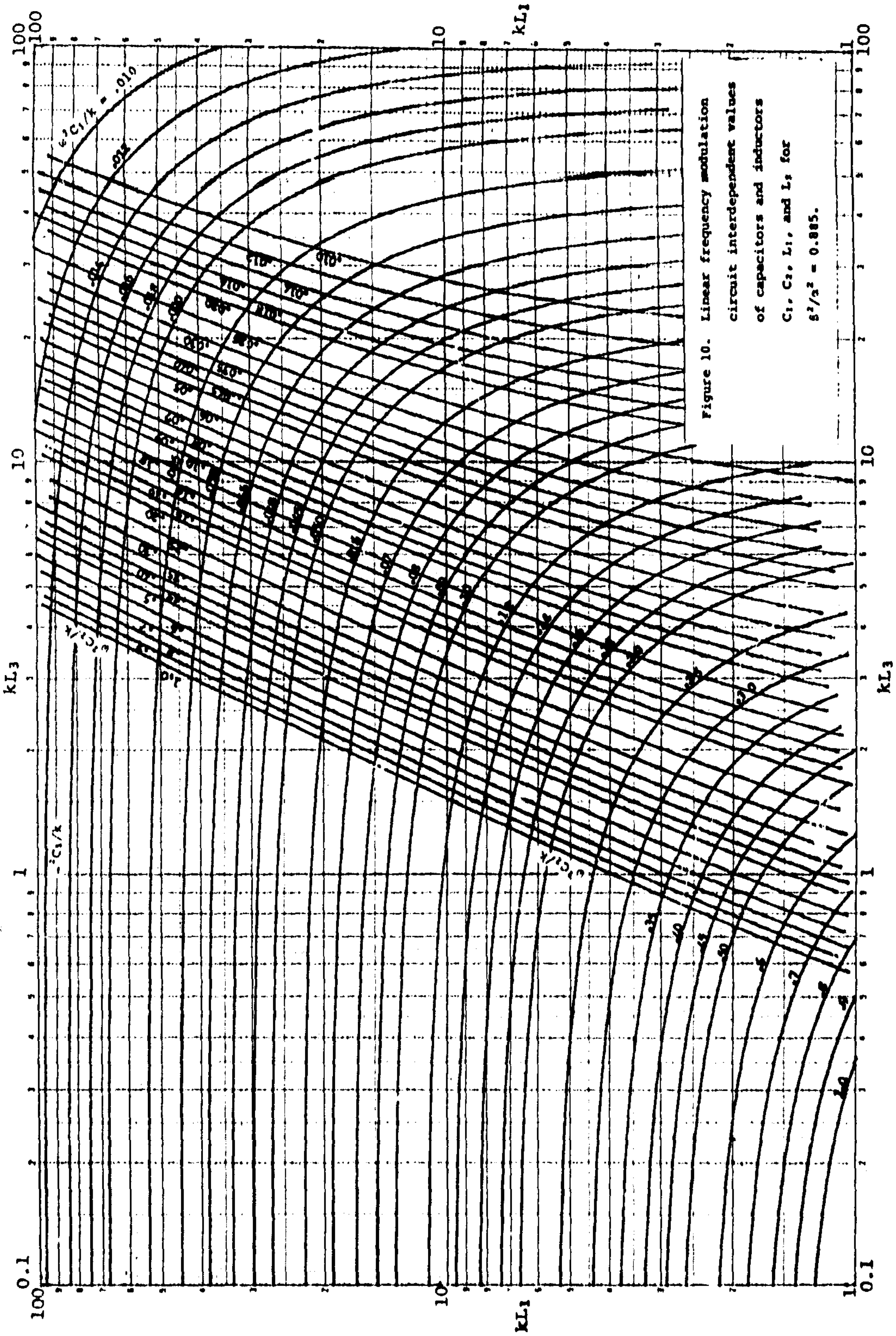

Figure 10. Linear frequency modulation circuit interdependent values of capacitors and inductors C_1, C_2, L_1, and L_3 for $\delta^2/s^2 = 0.885$.

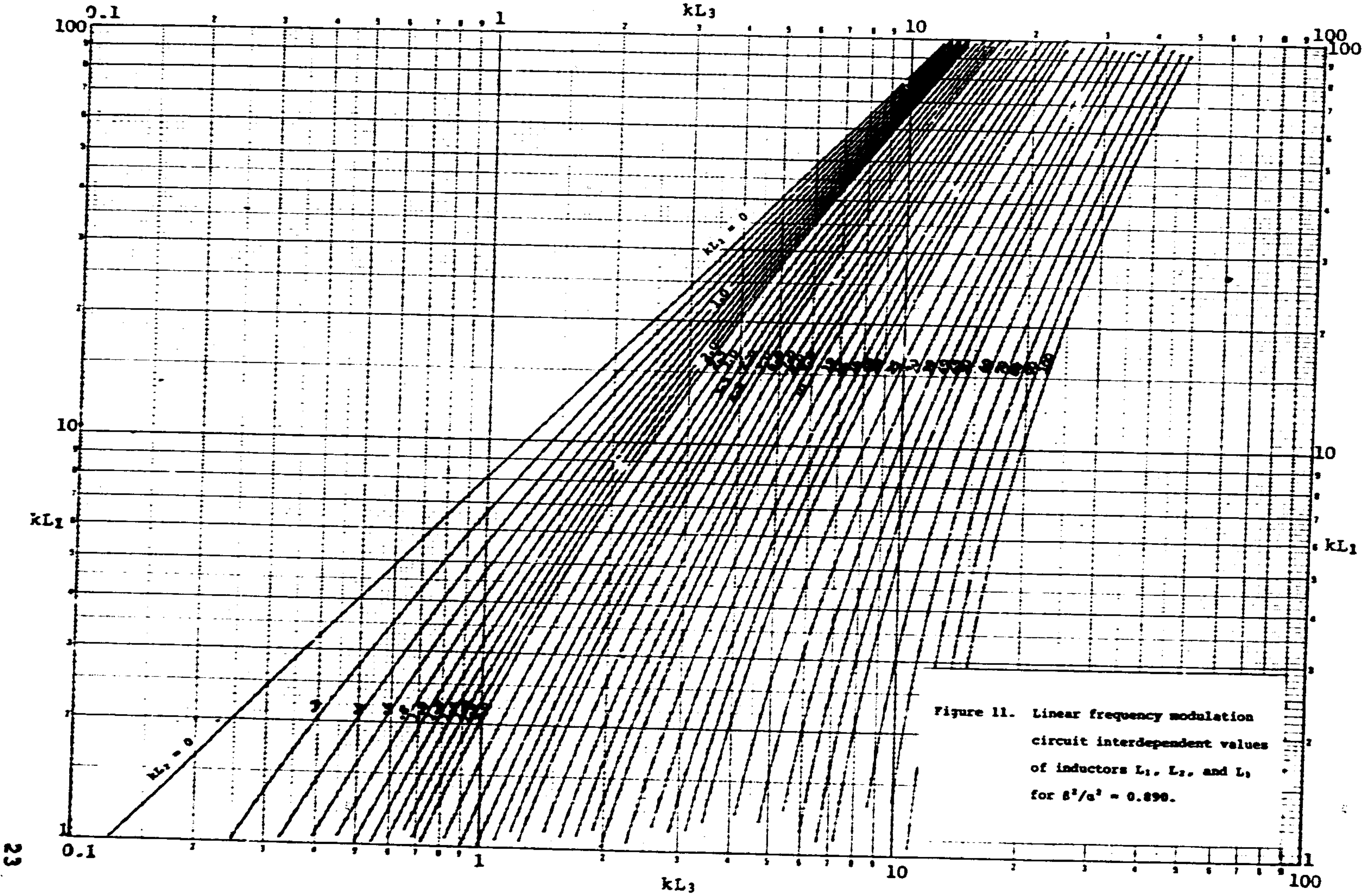

Figure 11. Linear frequency modulation circuit interdependent values of inductors L_1, L_2, and L_3 for $\delta^2/\alpha^2 = 0.890$.

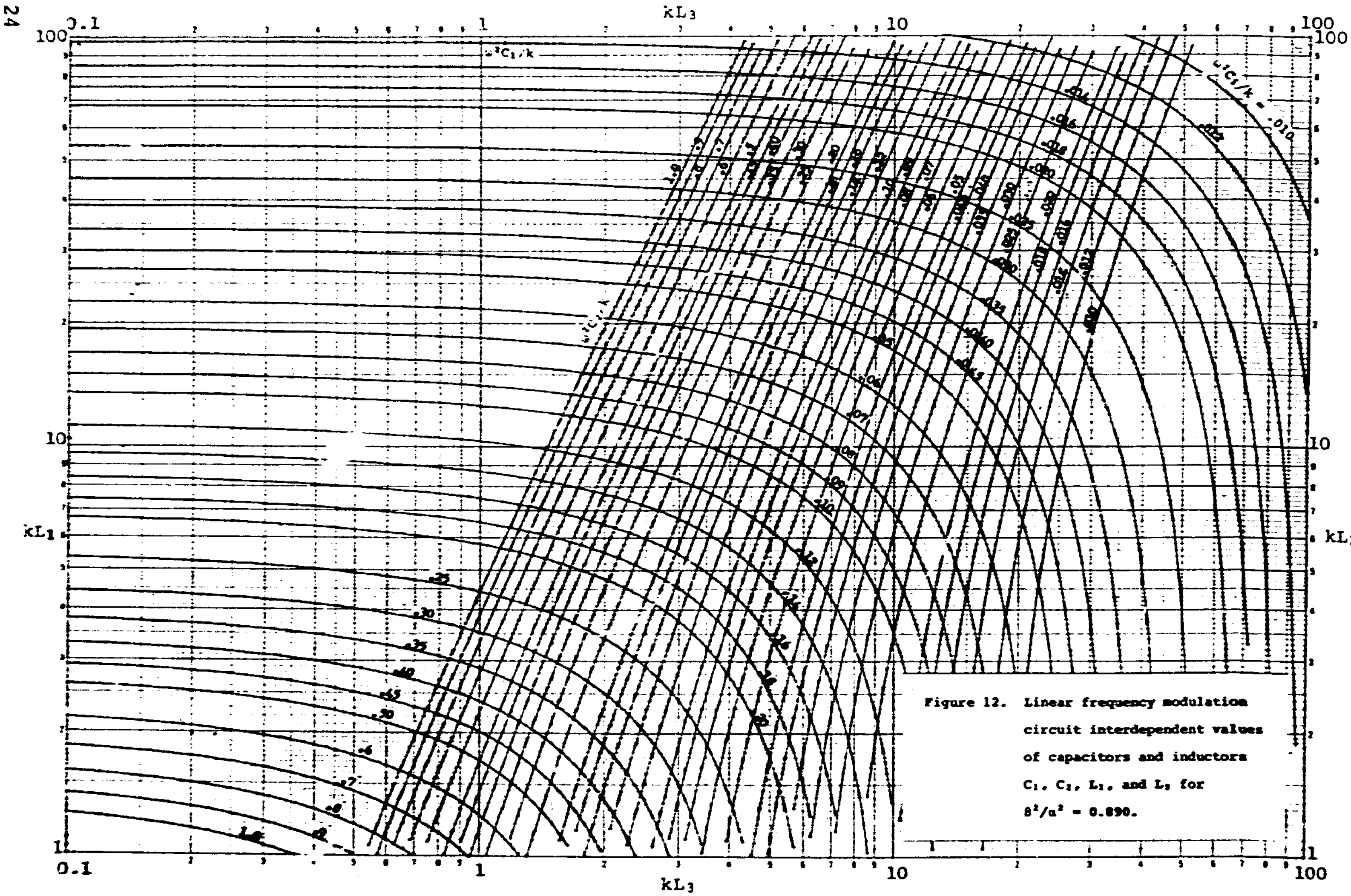

Figure 12. Linear frequency modulation circuit interdependent values of capacitors and inductors C_1, C_2, L_1, and L_3 for $\beta^2/\alpha^2 = 0.890$.

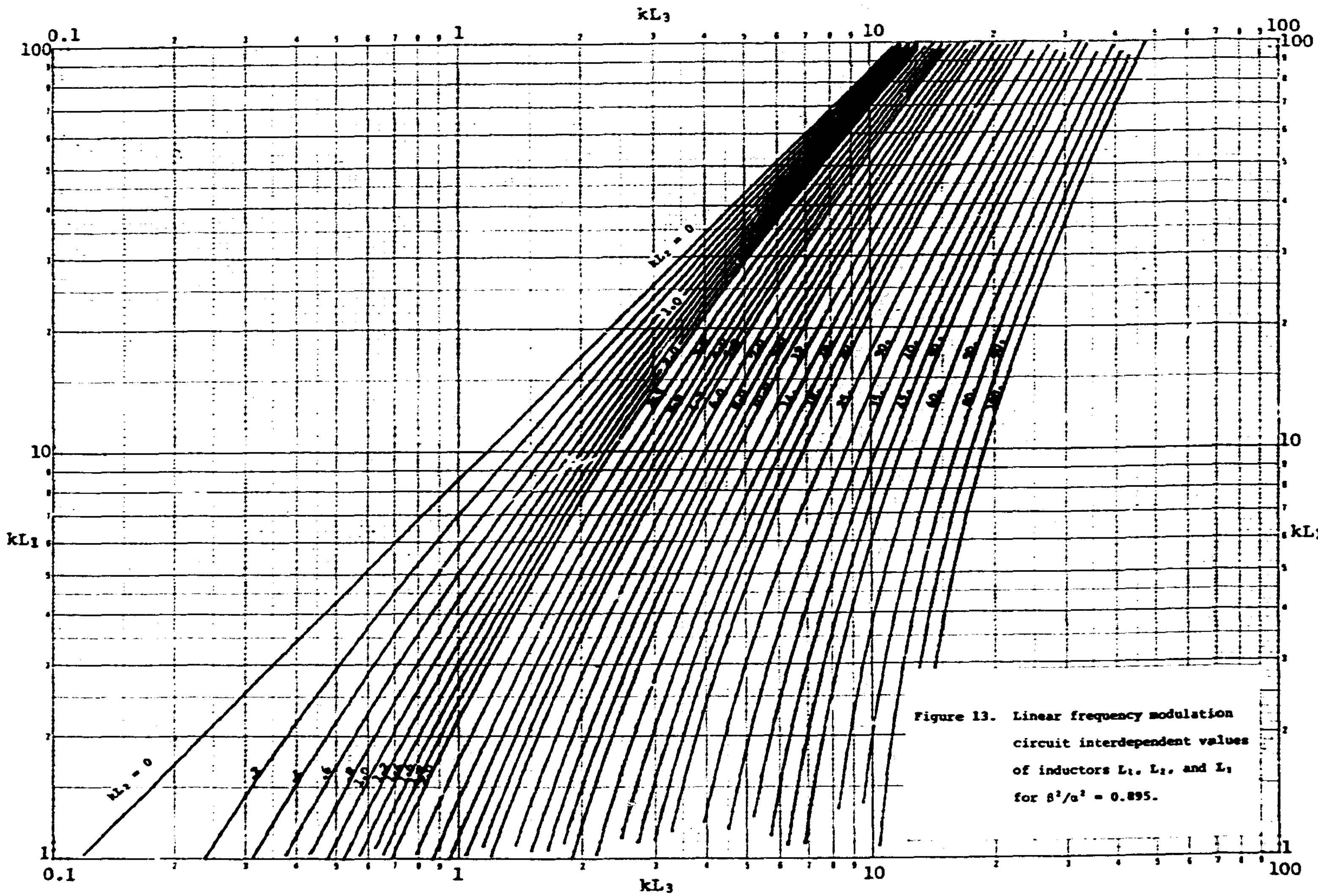

Figure 13. Linear frequency modulation circuit interdependent values of inductors L_1, L_2, and L_3 for $\beta^2/\alpha^2 = 0.895$.

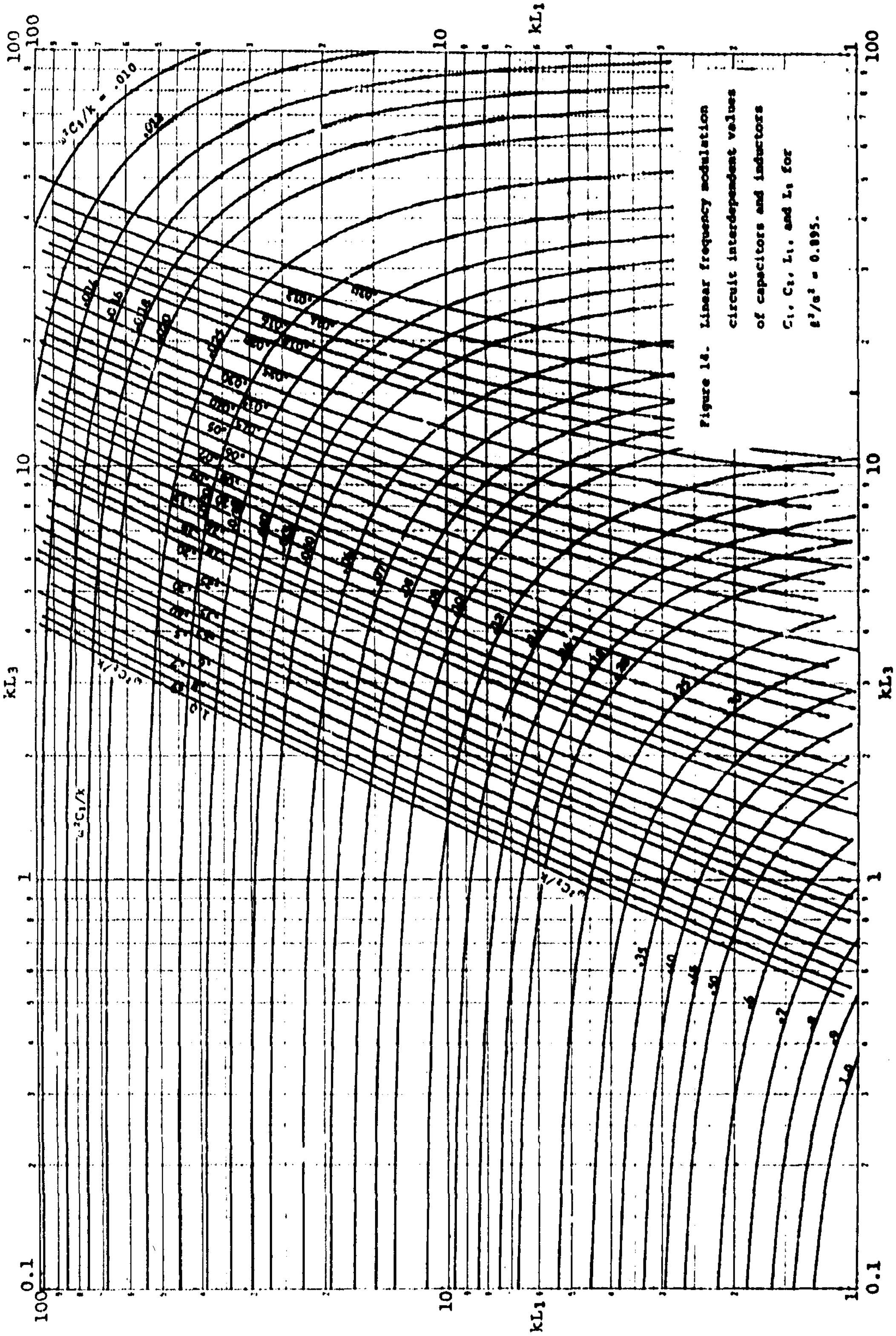

Figure 14. Linear frequency modulation circuit interdependent values of capacitors and inductors C_1, C_2, L_1, and L_3 for $f^2/f^2 = 0.895$.

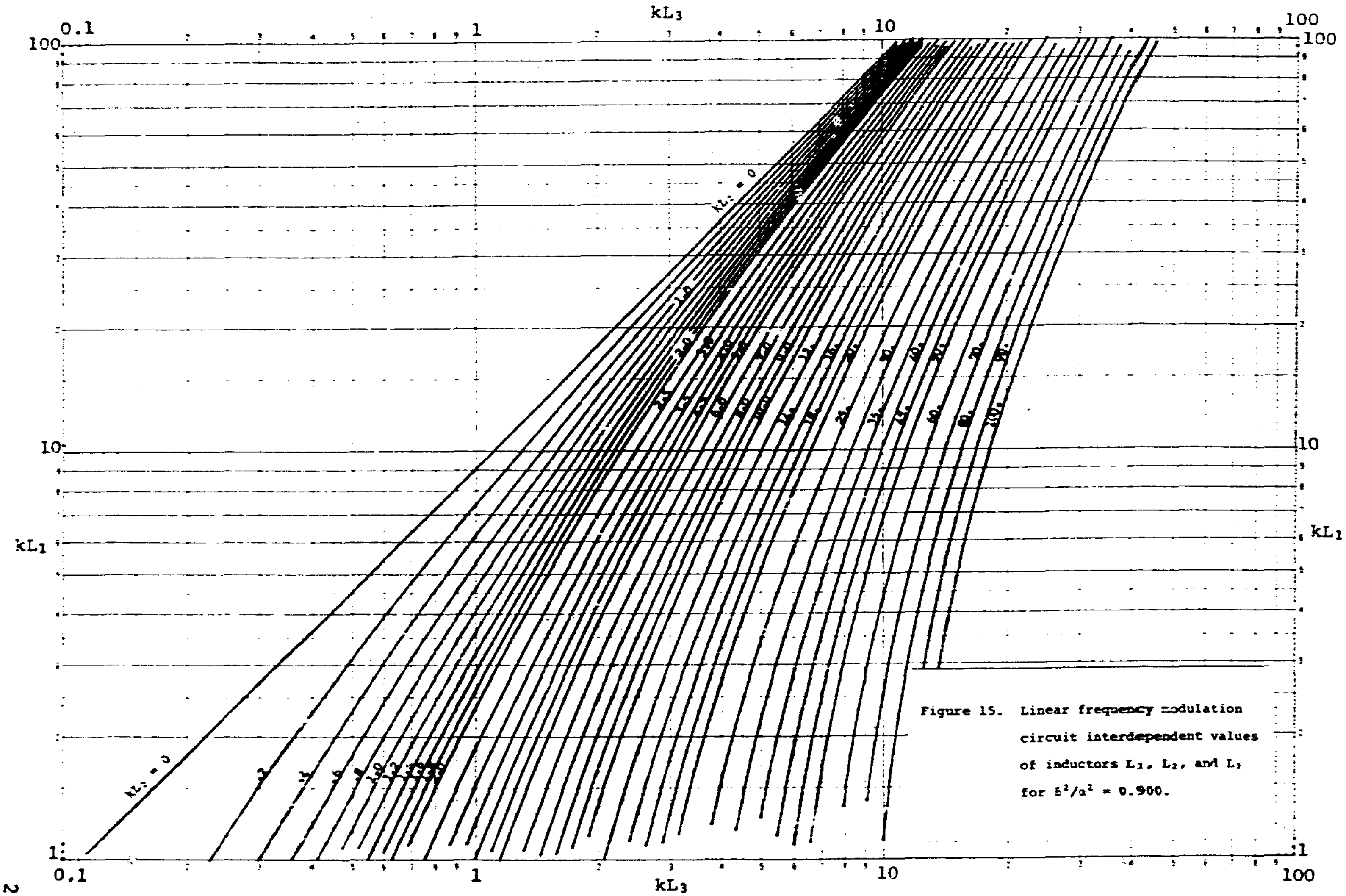

Figure 15. Linear frequency modulation circuit interdependent values of inductors L_1, L_2, and L_3 for $\xi^2/\alpha^2 = 0.900$.

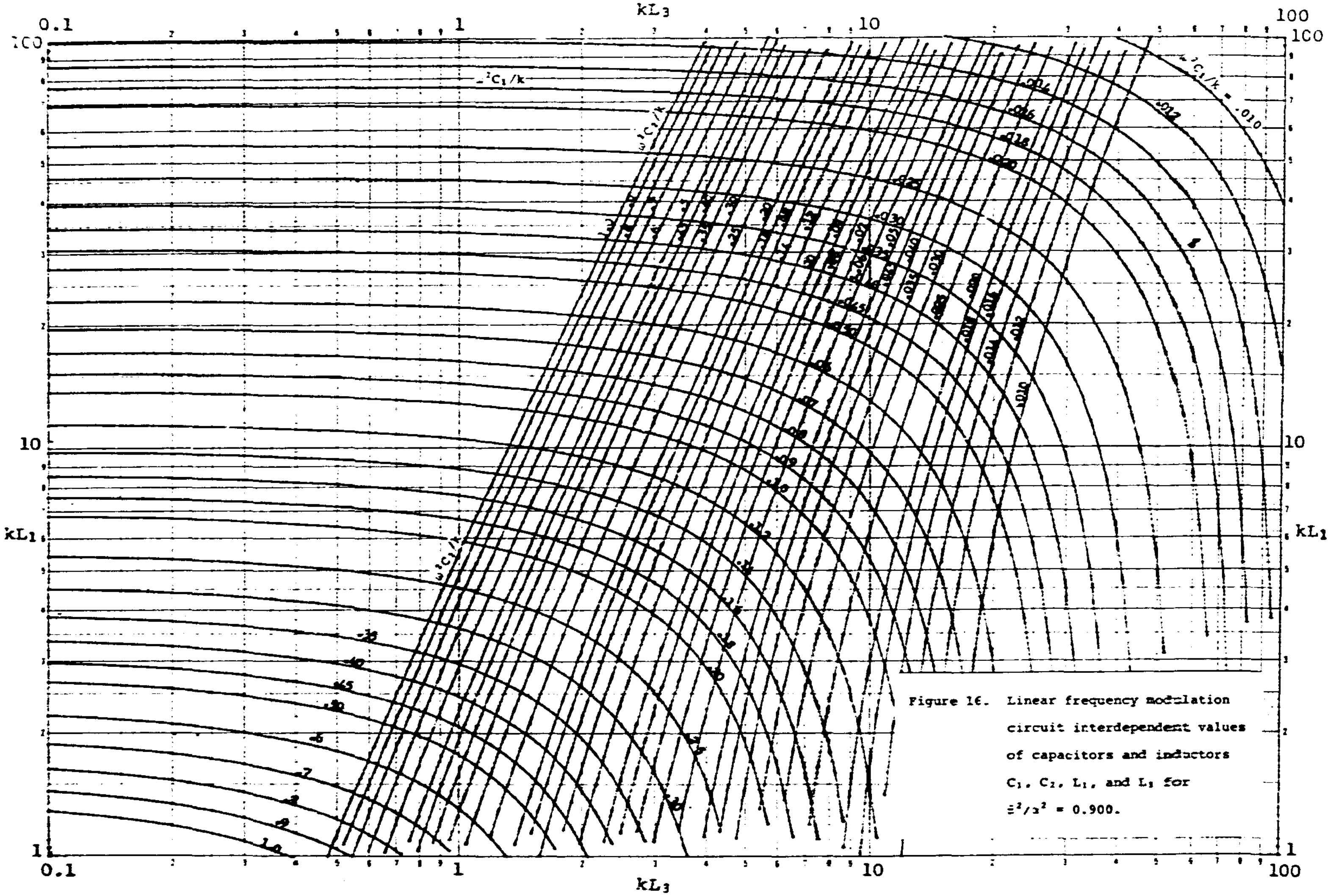

Figure 16. Linear frequency modulation circuit interdependent values of capacitors and inductors C_1, C_2, L_1, and L_3 for $\xi^2/a^2 = 0.900$.